**Market Microstructure Analysis on Turkish Stocks**

1. **Introduction**

In this project, Kyle et al. (2011) for the Turkish stock market. The market invariance hypothesis suggests that the distributions of risk transfers and transaction costs are constant when measured per unit of business time, that is, they depend on market velocity, that is, the speed at which new orders arrive at the market. To estimate the model, order and transaction book data for the Turkish stock market are intended to be used to answer the following questions.

It is observed that the frequency of transactions in financial markets has increased significantly due to developments in the field of high-frequency and algorithmic transactions. In order for a trader to get the maximum benefit from a trade, he must take into account price targets as well as his own market impact through the change in the price of the orders. The magnitude of this market impact may result in significant liquidity costs depending on the liquidity conditions prevailing in the market.

Various models have been proposed in the literature to estimate the market impact of transactions. An important paper in this literature is Kyle et al. (2016). This article introduces the concept of business time and Market Invariance Hypothesis (MIH). The authors argue that the distributions of risk transfers are constant when measured in units of job time. This is because business cycles occur at different rates for different stocks. For actively traded stocks, the business period is fast, while for inactive stocks, the business period is slow. As a result, the authors suggest that embedding job duration into market impact models can help eliminate their stochastic nature and increase the accuracy of these models.

The idea of work time or time-varying continuous-time stochastic processes was introduced by Mandelbrot and Taylor (1966) and Clark et al. (2003) and was introduced to the finance literature. Mandelbrot and Taylor (1966) manipulated time based on transactions. On the other hand, Clark et al. (2003) used volume, which is commonly used in the market microstructure literature. In the idea of working time, the basic structure of time is intact. In calendar time, each daily period consists of 24 hours. But in business time, each period contains equal amounts of a stochastic variable. For example, a period in a volume hour contains $1 million in volume. So, for some articles, a period of volume hours equals a quarter day, and for some articles, it equals five days. These numbers can be chosen differently. From another perspective, suppose there are two stocks, one of which transfers five times more shares of stock than the other during some period of calendar time. Let the first one be called swift stock and the second one be called slug stock. If transaction amount is measured in time, time for swift stock is five times faster than for slug stock.

1. **Relevant Methodology and Used Methods**

The mathematical aspect of time-changing Brownian motion was studied by Bochner (1949), Dubins (1965), Hunt (1957), Ito (1965). They noted that determinable time may follow a martingale or stochastic movement when interacting with a non-positive and non-decreasing random variable. The random variable must be non-positive and non-decreasing because time cannot be negative and cannot go backwards.

Clark (1973) was the first to use the idea of business duration in finance theory. He modeled cotton futures contract prices with Bochner's sub-stochastic process. In his study, he concluded that the subdistribution fits the data better than the standard distributions. After this study, work time has become an important alternative, especially when the research question is to model a stochastic structure.

Easley et al. (2012) showed that it is important to define the volume hour in terms of transaction volume. In this study, they emphasize that high-frequency transactions (HFT) have become important in financial markets, and argue that since HFT is carried out by machines, financial models should be based on algorithmic thinking when creating them. Since the machines' own clocks are event-based, not chronological, they calculate in cycles. Therefore, they demonstrated that using event-based cycles is more effective than using standard calendar time.

Kyle et al. (2016) proved the Market Invariance Hypothesis using bets. In this context, betting means transferred risk. They used portfolio change data to test their hypotheses. He also showed that transaction costs are invariant across stocks when measured in labor hours. In the same article, Kyle et al. (2016) developed a liquidity model that uses business time as the main unit of time for transactions. Kyle's methodology provides an important structure for building a market impact model, as liquidity is one of the main determinants of price impact and the model reflects relationships between market variables and price.

Andersen et al. (2018), Kyle et al. (2016) developed the Intraday Trade Invariance model. He explained the logarithmic volatilities of mini S&P 500 futures contracts using the logarithm values of the transaction amount and transaction size. Kyle et al. (2016), Andersen et al. (2018) used transaction data instead of portfolio change data and showed that market invariance can be achieved not only with portfolio change data but also with order book data.

For Turkey, Ersan and Ekinci (2021) worked with order book data to analyze the impact of news announcements. It has shown that the reaction to news announcements is slower than in developed markets and that event-driven strategies offer profit opportunities in the Turkish stock market. In their previous studies, Ersan et al. (2021), again using order book data, revealed that HFT participation in the Turkish market is low.

1. **Scientific Finds and Results**
   1. **Dataset**

Borsa Istanbul has switched its data transmission system to BISTECH in accordance with global industry standards. Within the BISTECH framework, data has a structure suitable for binary coding, which significantly increases the processing speed compared to the previous system. BIST organizes incoming data into separate chunks, keeping it in a non-concatenated format to protect access to raw data. Therefore, it is necessary to first make the raw data suitable for analysis.

The data used in this project comes from two primary sources: (i) order book data, which forms the limit order book, and (ii) transaction information, which forms the transaction book.

There are 32 columns in the order book data presented monthly, and we can divide these columns into three groups: identifiers, feature and detail. There are an average of 50 million observations in a month's data. Descriptive columns are used to distinguish orders from each other and sort them accordingly. These descriptive columns include: Date, Order Number, Order Entry Date and Time, Order Modification Date and Time, Transaction Code, Buy/Sell, Update Number, Update Time.

The date column holds the day the order was recorded and contains Order Number, System Opening Time, Partition Information and a singular value. It should be noted that this column does not contain any information about the order of the order. Order Entry Date and Time shows the date and time the Order was modified. There is a space character between date and time. The order in the entire order book is based on the value in this column. The transaction code indicates which instrument the order belongs to and includes the serial code of that instrument. The Buy/Sell column indicates whether the order is in the down direction or in the sell direction. An order's status often changes due to both the transaction and the order owner's updates. We track such changes through the Update Number. Finally, Update time indicates when the said update took place.

Our second group columns are as follows: Order Price Type, Order Category, Order Validity Round, Order Status, Order Change Reason, Order Quantity, Remaining Quantity, Price, Session, Best Buy Price, Best Sell Price. Order Price Type indicates the type of order sent and includes examples such as Limit order, market order, market to limit order. Since Limit Order Book is used in our study, Limit order types are emphasized. Order Category is a column that indicates whether the order is sent as an order, quote or special transaction notification. We only use orders. Order Validity Round is a column that expresses how long the order will remain active; Examples include daily or until canceled. At the time we examined, orders generally consisted of daily orders.

Order Status indicates whether the sent order is up to date or not. This column indicates orders that become inactive for various reasons before the lot amount on them is exhausted. So, this column is an important variable that helps us determine which orders we should buy and take when taking snapshots of the Limit Order Book. Order Change Reason is a column that explains the reason for certain changes. For example, reasons such as cancellation by the system, cancellation by the user, transaction etc. can be observed.

Order Quantity is a variable that indicates the amount of lots the order carries when it is entered. Remaining Quantity shows the remaining lots after the changes, so it is an important column. Price shows the current price of the relevant order. Session gives information about the session to which the relevant order belongs. Best Buy Price and Best Sell Price provide information about the best buy and best sell prices after the order is entered, respectively.

The third group of columns contains detailed information that is not actively used in this study. Their names are as follows: Order Type, Display Amount, Agency Code, Number of Transactions, Previous Order Number, Joint Contract Number, Giveup Flag.

Transaction Book is a ledger that records market orders where limit orders meet and turn into transactions. It is reported monthly like the Order Book, but there is no aggregation and access to raw data is provided. There are 21 columns in this ledger and it is used in our study to measure the trading volumes and liquidity of Shares on a lot basis.

By using columns named Date, Transaction Code, Buy/Sell, Order Number, Transaction Time to define each order, we can track individual orders in the ledger and group them according to their characteristics. Number of Transactions, Transaction Price, Transaction Volume columns report us the lot amount of the transaction and at what price it was passed. The Transaction Volume column is obtained by multiplying the Transaction Number and Transaction Price variables.

Remaining Joint Agreement Number, Member Transaction Number, Normal Transaction Notification, Session, Transaction Status, Clearing Date, Active/Passive, Clearing Party Transaction Number, Clearing Party Joint Agreement Number, Transaction Change Date and Time, Giveup Flag, Update Number, Update Columns named Time are columns that contain detailed information that is not used in our study.

* 1. **Results**

There are two basic mechanisms we need to establish in the project to test the Market Invariance hypothesis. The first is to create a limit order book that will display the raw data in BISTECH format in real time at any time. Secondly, it is to calculate to what extent the prices will change (market impact) when a designed order is sent to this order book.

When we conducted the study using the first mechanism, the previously mentioned orders were sent to the instant sections taken from the limit order book at 5-minute intervals for each share. The amounts of these artificial orders sent for both buying and selling are calculated dynamically. Artificial orders were created using percentage of volume, which is widely used in the literature. Here, percentage volume refers to the daily transaction volumes that occurred in the past. The transaction book we mentioned in the previous section was used when creating the volumes. In addition, in order for the dynamic order amount calculation method to work more smoothly, the daily volume is calculated using the average of the daily volumes of the last 20 trading days. Since we expected the same limit order book to behave differently in different amounts of orders, orders were sent for 0.5%, 1% and 2% of the daily transaction volume. In addition, the middle price between the best buying price and the best selling price before the order was sent was calculated and recorded. To measure liquidity, the bid-ask spread was calculated. Afterwards, the new price formed by the second mechanism was calculated. It is possible to calculate the change in price (Market Impact) in two ways. The first is calculated using the percentage change of the price, as the percentage change relative to the median price, as observed in the literature and real-life applications. The second method is price change in price ticks. For this method, it is determined by measuring how far the newly formed price moves from the middle price (the previously calculated buying and selling price spread). While the first method is useful for more practical analyzes such as liquidity cost, the second method is more advantageous for comparing stocks with different liquidities.

Table 1

A table of stock numbers

Description automatically generated

The percentage change and market impact calculation values for selected stocks are shown in Table 1. The analysis was conducted for 409 stocks in the data set, but due to space constraints, values are presented in Table 1 only for the 5 most and least liquid stocks. It is observed that liquid stocks selected according to the transaction book have lower market impact, as expected. Here the price change is calculated for 1% volume. The cross-section average was found by averaging the values of price changes in the same section over all days. Then, after obtaining the average values in each section, the median of these values was selected to represent the day.

The liquid shares of the selected shares are respectively: Afyon Çimento Sanayi T.A.Ş., Emlak Konut Gayrimenkul Yatırım Ortaklığı A.Ş., Anadolu Cam Sanayii A.Ş., Alkim Alkali Kimya A.Ş., ACISELSAN Acıpayam Selüloz Sanayi ve Ticaret A.Ş.

The relevant ones are as follows: Avrasya Petroleum and Touristic Facilities Investments Inc., RTA Laboratories Biological Products Pharmaceuticals and Machinery Industry Trade Inc., PARK Electricity Production Mining Industry and Trade Inc., Avrasya Gayrimenkul Yatırım Ortaklığı A.Ş. ., Alcatel Lucent Teletaş Telekomünikasyon A.Ş.

Figure 1

A graph of a number of timestamp

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Figure 1 presents the market effects calculated with the step count method. BIMAS.E (BIM United Stores A.Ş.) stock, which is relatively liquid and had median liquidity at the time studied, was selected. The price tick of the stock at the time examined is 5 cents. The data presented in Figure 1 was calculated by averaging the sections of the same moment of each day. For example, the 11.05.00 time section is the average of all days' 11.05.00 section. Thus, the market effect resulting from orders of three different amounts can be examined at different times of the day. While a high effect is observed in both directions on the relevant stock in the morning hours, the effect becomes stagnant later in the day. In addition, it is possible to say that the sell order has less impact on this stock. Therefore, it can be stated that the buy limit order book shows higher market depth than the sell order book.

Figure 2

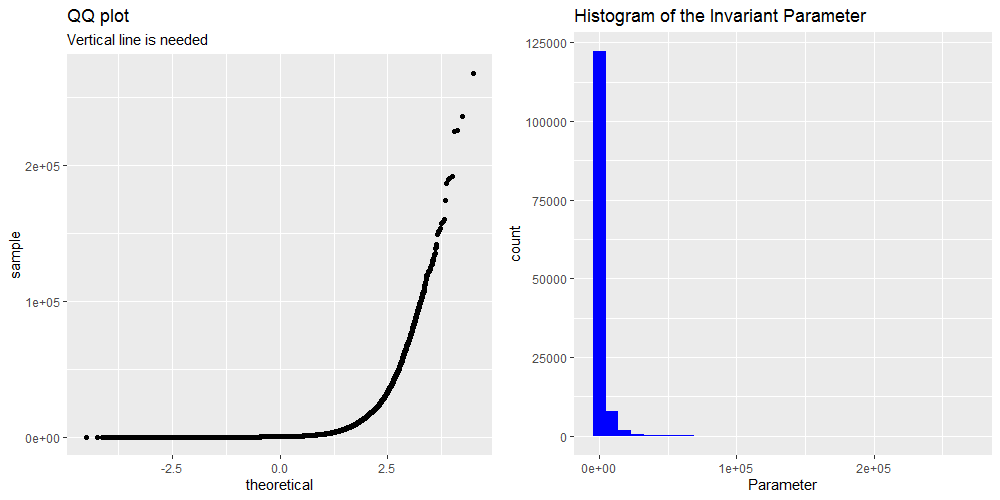
A graph of different colored lines

Description automatically generated

Figure 2 shows the entire market analysis, which is the next stage of the analysis presented in Figure 1. Here, the individual stock analysis results were averaged and aggregated for each stock traded on Borsa Istanbul, and the market impact was determined. Thus, it was analyzed how BIST was exposed to market effects during the day.

It is observed that the price change is high in both directions at the opening of the morning session. During the lunch break, a collective decrease is observed in the changes due to the effect of the stocks in the markets where the single price method is used in BIST. When examined carefully, an increase in market influence is observed at the opening of the noon session, although less than in the morning. Except for the mentioned cases, it is observed that the market effect for the entire BIST remains stable at all order amounts.

Figure 3



In this part of the project, the market invariance hypothesis mentioned in Kyle et al.'s (2016) study was tested on all BIST stocks. Kyle expresses his hypothesis with the following equation:

Here, I represents the invariance parameter, Pj represents the stock for time t, Q represents the expected order amount in lots for j shares at time t, and represents the expected price volatility for j shares and time t. Finally, share j represents the expected speed for time t. Speed is considered as the number of orders received at the mentioned time.

To carry out this analysis, the transaction book was used, and the relevant variables were created by calculating the total amount, average price, total number of orders and price volatility in the section for fifteen-minute sections from the book, which has an average of 20 million monthly observations. Then, the past 20 days were used for expected values and the 20-day moving average was shifted back one day to avoid look-ahead bias. Then, the above-mentioned equation was created and the invariant market parameter was calculated for each stock and each cross-section. As we see in Figure 3, the distribution of the parameter is skewed to the right and can have extreme values on the positive side. In addition, in QQ images, a near-horizontal line should be formed in variables that are constant or almost constant, but in our case, a horizontal line cannot be observed. These findings show that the Market Invariance Hypothesis did not work in the time period we investigated.

1. **Evaluating the Results**

In this project, Kyle et al. (2011) for the Turkish stock market. To estimate the model, order and transaction book data for the Turkish stock market are intended to be used to answer the following questions.

First of all, a market impact analysis was conducted in terms of buying and selling for stocks traded on BIST, and the average market impact was calculated for all stocks traded on BIST. After this analysis, how the market impact changes at certain times during the day is examined for a single sample stock (BIM stock with median liquidity) and the entire BIST.

At the opening of the morning session, it was observed that price changes were high in both directions. During the lunch break, a collective decrease in price changes was determined, with the effect of the stocks in the markets using the single price method in BIST. At the opening of the noon session, there is an increase in market influence, although less than in the morning. Except for these cases, it has been shown that the market impact for the entire BIST is generally stable across all order amounts.

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